ELEN 4810 Final Exam

Monday, December 18, 2023, 1:10-4:00 PM. Two sheets of handwritten notes are allowed. No electronics of any kind are allowed. Please record your answers in the exam booklet. Raise your hand if you need additional scratch paper.

There are a total of 4 questions. Good luck!

Name:

Uni:

1. ***Z*-transform.** A discrete-time LTI system has transfer function

$$H(z) = \frac{(1+\frac{3}{2}z^{-1})(1-\frac{3}{2}z^{-1})}{(1+j\frac{1}{2}z^{-1})(1-j\frac{1}{2}z^{-1})}. \tag{1}$$

Please answer the following questions:

Part 1. Please plot the pole-zero diagram of $H(z)$, labeling all poles and zeros. You can use the axes on the next page.

Part 2. Which of the following best describes this system?

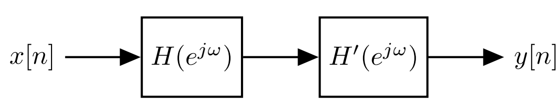
HIGH PASS   LOW PASS   ALL PASS   BAND PASS

Please justify your answer.

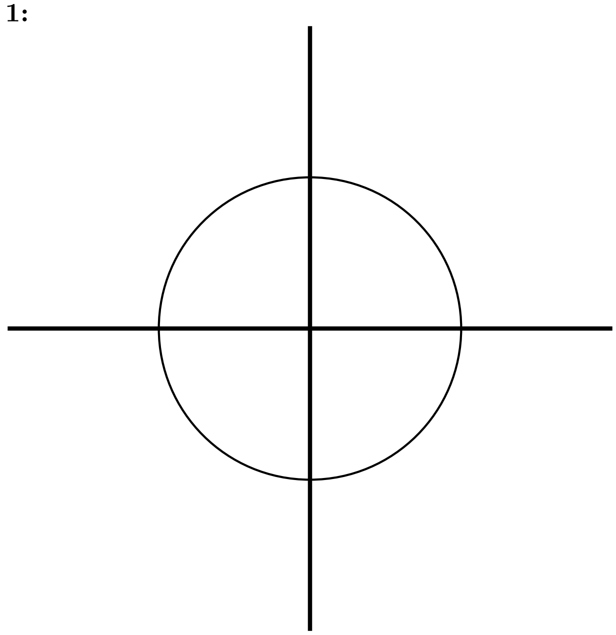
Part 3. Could the system be stable and causal? Why or why not?

Part 4. Could the impulse response $h[n]$ be real valued? Why or why not?

Part 5. Please determine the transfer function $H'(z)$ of a system which is **causal** and **stable**, *and* ensures for the following system which applies H and H' in series, the output $y[n]$ satisfies $|Y(e^{j\omega})| = |X(e^{j\omega})|$ **for every input** $x[n]$:



Answer to Problem 1:



2. Generalized Linear Phase Systems. Consider an FIR generalized linear phase system, with real valued impulse response $h[n]$, transfer function $H(z)$ and zeros $\zeta_1, \dots, \zeta_{L-1}$.

Please answer the following questions:

Part (i). What are the poles of $H(z)$? For any repeated poles, please indicate their multiplicity. If it is not possible to determine the poles from the given information, please explain why.

Part (ii). Set $h'[n] = (-1)^{n+1} h[n]$. Does the resulting system have generalized linear phase? Why or why not?

Part (iii). Please give an expression for the zeros $\zeta'_1, \dots, \zeta'_{L-1}$ of $H'(z)$ in terms of $\zeta_1, \dots, \zeta_{L-1}$.

Part (iv). Suppose we wish for $h'[n]$ to be a **low pass** system. What types of canonical generalized linear phase system/systems should we *not* choose for $h[n]$? Why?

Answer to Problem 2:

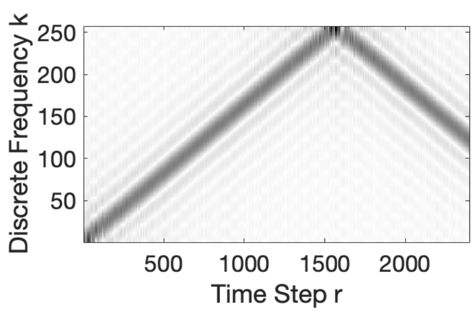
3. Spectrograms. A continuous-time chirp signal

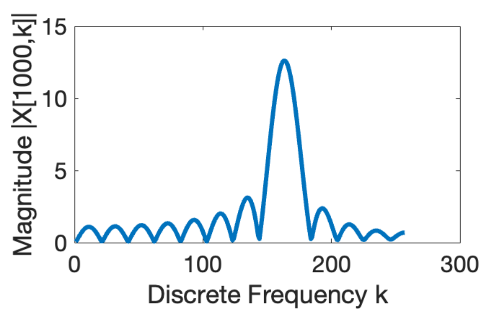
$$x_c(t) = \cos(\alpha t^2)$$

is sampled with a sampling period

$$T_s = 0.03 \,\text{seconds}$$

to produce a discrete time signal $x[n]$. We compute the Short-Time Fourier Transform (STFT), $X[r, k]$, using a time stride of $R = 10$ samples, $N = 512$ frequency samples, and a window $w[n]$ of length L. We plot the magnitude of the Short-Time Fourier Transform (STFT) $|X[r, k]|$, for $r = 0, \ldots, 2400$ and $k = 0, \ldots, 255$:





Above, the graph at right plots the vertical slice

$$X[1000, 0], X[1000, 1], \ldots, X[1000, 255].$$

(Note, that $N = 512$; here we only show $X|[r, k]|$ for $k < N/2$).

Please answer the following questions:

Part (a). Please estimate the chirp parameter α. Justify your answer!

Part (b). Why does the spectrogram exhibit a rising line and a falling line?

Part (c). Which of the following windows was used to determine the spectrogram?

RECTANGULAR HAMMING

Please explain your answer!

Part (d). Please estimate the length L of the window, based on the available information.

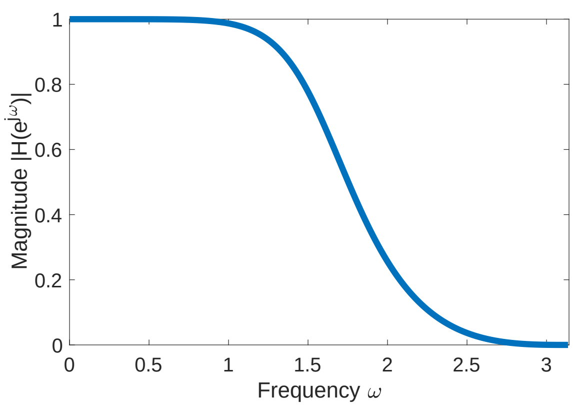
Note: your estimate does not need to be perfect, but please explain how you arrived at it.

Answer to Problem 3:

4. IIR Filter Design and Bilinear Transform. We generate a discrete time IIR filter by applying bilinear transformation

$$H(z) = H_c\left(\frac{z-1}{z+1}\right)$$

to a continuous time system, with transfer function $H_c(s)$. Here is the magnitude response $|H(e^{j\omega})|$:



Please answer the following questions:

Part A. Which of the following best describes this filter?

BUTTERWORTH CHEBYSCHEV I CHEBYSCHEV II ELLIPTIC

Part B. Our continuous time filter $H_c(s)$ has poles at

$$s = -1, \quad s = \exp\left(\frac{j2\pi}{3}\right), \quad s = \exp\left(-\frac{j2\pi}{3}\right),$$

and a zero of multiplicity three at $s = \infty$. **What are the poles and zeros of the discrete time system $H(z)$?**

Part C. What is one advantage of FIR filters (e.g., designed by windowing or optimization) compared to the IIR design in part A?

Part D. Suppose we generate a new system, by setting $H'(z) = H(z)H^*(1/z^*)$. **What is the phase response $\angle H'(e^{j\omega})$ of the new system?**

Part E. The system H' has order six (six poles and six zeros). Based on your answer to Part D, please describe one advantage to the system H', compared directly designing an order six system using bilinear transformation.

Part F. Can the system $H'(z)$ be both **stable** and **causal**? Why or why not?

Part G. Please give an expression for the impulse response $h'[n]$ in terms of the impulse response $h[n]$.

Answer to Problem 4: